

A Giant Halo at the Neutron Drip Line¹

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Relativistic Hartree-Bogoliubov (RHB) theory in coordinate space is used to describe the chain of even-even Zirconium isotopes reaching from ^{116}Zr to the drip line nucleus ^{140}Zr . Pairing correlations are taken into account by a density dependent force of zero range. For neutron numbers larger than the magic number $N = 82$ a giant neutron halo outside the core of ^{122}Zr is observed. It is formed by up to six neutrons.

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With perpetual improvement in the techniques of producing radioactive beams, the study of exotic nuclei very far away from the line of β -stability has now become feasible. Experiments of this kind may cast new light on nuclear structure and novel and entirely unexpected features may appear: Neutron rich nuclei can have a structure very different from that of normal nuclei. They consist of a normal core surrounded by a skin of neutron matter. Close to the drip line, where the coupling to the continuum becomes important, a neutron halo can develop, as it has been observed in several light nuclei, the most famous being ^{11}Li [2]. However, in all of the halos observed so far, one has only a very small number of neutrons, namely one or two outside of the normal core.

In order to study the influence of correlations and many-body effects it would be very interesting to find also nuclei with a larger number of neutron distributed in the halo. In this letter we report on the theoretical prediction of a giant neutron halo for Zr-isotopes close to the neutron drip line. It is formed by up to six neutrons outside of the ^{122}Zr core with the magic neutron number $N = 82$.

Recently, a fully self-consistent relativistic method has been developed for the description of halo nuclei. It is based on Relativistic Hartree-Bogoliubov (RHB) theory in coordinate space [3]. It is able to take into account at the same time the proper isospin dependence of the spin-orbit term, which is very crucial for a reliable description nuclei far away from the line of β -stability [4] and the self-consistent treatment of pairing correlations in the presence of the continuum [5]. As it has been shown in Ref. [3] the scattering of Cooper pairs into the continuum containing low-lying resonances of small angular momentum plays an important role for the formation a neutron halo. By using a density dependent zero range interaction, the halo in ^{11}Li has been successfully reproduced in this self-consistent picture. Very good agree-

ment with recent experimental data is obtained without any adjustment of parameters. To obtain these results, a full solution of the RHB equations [6] in coordinate space is necessary. The expansion in a harmonic oscillator basis [7], which is very useful only for nuclei close to the line of β -stability, provides only a very poor approximation in the continuum, even if a large number of oscillator shells is taken into account. The simple BCS-approximation used in Ref. [4] leads to a partial occupation of unbound states in the continuum and therefore to a gas of evaporating neutrons. Here we report on calculations, where the same method as in Ref. [3] has been used to investigate halos in Zr-nuclei at the neutron drip line.

The starting point is the Relativistic Mean Field Lagrangian

$$\begin{aligned} \mathcal{L} = & \bar{\psi} \left(\not{p} - g_{\omega} \psi - g_{\rho} \vec{\rho} \vec{\tau} - \frac{1}{2} e (1 - \tau_3) A - g_{\sigma} \sigma - m \right) \psi \\ & + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - U(\sigma) - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_{\omega}^2 \omega_{\mu} \omega^{\mu} \\ & - \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_{\rho}^2 \vec{\rho}_{\mu} \vec{\rho}^{\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \end{aligned} \quad (1)$$

It describes the nucleons with the mass m as Dirac spinors ψ moving in the fields of mesons: a scalar meson (σ), an isoscalar vector meson (ω), an isovector vector meson ($\vec{\rho}$) and the photon A^{μ} , with the masses m_{σ} , m_{ω} and m_{ρ} and the coupling constants g_{σ} , g_{ω} , g_{ρ} . The field tensors for the vector mesons are given as $\Omega_{\mu\nu} = \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu}$ and by similar expressions for the ρ -meson and the photon. For simplicity we neglect these fields in the following, however, they are fully taken into account in the calculations. For a realistic description of nuclear properties a nonlinear self-coupling $U(\sigma) = \frac{1}{2} m_{\sigma}^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$ for the scalar mesons has turned out to be crucial [8].

Using Green's function techniques it has been shown in Ref. [6] how to derive the relativistic Hartree-Bogoliubov (RHB) equations from such a Lagrangian:

$$\begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix}_k = E_k \begin{pmatrix} U \\ V \end{pmatrix}_k, \quad (2)$$

E_k are quasiparticle energies and the coefficients $U_k(r)$ and $V_k(r)$ are four-dimensional Dirac spinors. h is the usual Dirac hamiltonian

$$h = \boldsymbol{\alpha} \mathbf{p} + g_{\omega} \omega + \beta (M + g_{\sigma} \sigma) - \lambda \quad (3)$$

containing the chemical potential λ adjusted to the proper particle number and the meson fields σ and ω de-

terminated as usual in a self-consistent way from the Klein Gordon equations in *no-sea*-approximation.

The pairing potential Δ in Eq. (2) is given by

$$\Delta_{ab} = \frac{1}{2} \sum_{cd} V_{abcd}^{pp} \kappa_{cd} \quad (4)$$

It is obtained from the pairing tensor $\kappa = U^* V^T$ and the one-meson exchange interaction V_{abcd}^{pp} in the *pp*-channel. More details are given in Ref. [6]. There it has been found, that these forces are not able to reproduce even in a semi-quantitative way proper pairing in the realistic nuclear many-body problem. As in Ref. [3] we therefore replace V_{abcd}^{pp} in Eq. (4) by the density dependent two-body force of zero range:

$$V(\mathbf{r}_1, \mathbf{r}_2) = V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \frac{1}{4} [1 - \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2] \left(1 - \frac{\rho(r)}{\rho_0} \right), \quad (5)$$

where $r = |\mathbf{r}_1 + \mathbf{r}_2|/2$.

The RHB equations (2) for zero range pairing forces are a set of four coupled differential equations for the HB Dirac spinors $U(r)$ and $V(r)$. They are solved for the parameter set NLSH [9], which is widely used for the description of neutron rich nuclei, by the shooting method in a self-consistent way. The details will be published elsewhere. With a step size of 0.1 fm and using proper boundary conditions the above equations are solved in a spherical box of radius $R \geq 15$ fm. The results do not depend on the box size for $R \geq 20$ fm. For a radius $R = 30$ fm we found the same results within an accuracy of 0.1 % for the binding energies and matter radius. Since we use a pairing force of zero range (5) we have to limit the number of continuum levels by a cut-off energy. For each spin-parity channel 20 radial wavefunctions are taken into account, which corresponds for $R = 20$ fm roughly to a cut-off energy of 120 MeV. For fixed cut-off energy and for fixed box radius R the strength V_0 of the pairing force (5) for the neutrons is determined by a calculation in the nucleus ^{116}Zr adjusting the corresponding pairing energy $-\frac{1}{2}\text{Tr}\Delta\kappa$ to that of a RHB-calculation using the finite range part of the Gogny force D1S [10]. For ρ_0 we use the nuclear matter density 0.152 fm^{-3} . In order not to miss any bound state the cut-off energy has to be larger than the depth of the potential. But as long as this is the case and as long as the interaction strength is properly renormalized, the results of this investigation stay practically unchanged. And in such a way, we get almost the same result from the the pairing force of zero range and the finite range Gogny force.

In the upper panel of Fig. 1 we show the *rms* radii of the protons and neutrons for the Zirconium isotopes with mass numbers $A = 110$ to $A = 140$, the drip line nucleus. They are given as

$$\langle r^2 \rangle^{1/2} = \left[\frac{1}{N_\tau} \sum_{lj} \langle r \rangle_{lj}^2 \right]^{1/2}, \quad (6)$$

where $N_\tau = Z, N$ for protons and neutrons and

$$\langle r \rangle_{lj} = \left(\int r^2 \rho_{lj} d^3r \right)^{1/2} \quad (7)$$

are the contribution of the different blocks with the quantum numbers l and j . To guide the eye we also give a dashed line with a $N^{1/3}$ -dependence. It clearly shows a kink for the neutron *rms*-radius at the magic neutron number $N = 82$.

These results can be understood more clearly by considering the microscopic structure of the underlying wave functions and the single particle energies in the canonical basis [11]. Therefore, in the lower panel of Fig. 1 we show the single particle levels in the canonical basis for the isotopes with an even neutron number as a function of the mass number. Going from $N = 70$ to $N = 100$ we observe a big gap above the $1h_{11/2}$ orbit. For neutron numbers larger than the magic number $N = 82$, the neutrons are filled to the levels in the continuum or weakly bound states in the order of $3p_{3/2}$, $2f_{7/2}$, $3p_{1/2}$, $2f_{5/2}$ and $1h_{9/2}$.

The neutron chemical potential is given in this Figure by a dashed line. It approaches rapidly the continuum already shortly after the magic neutron number $N = 82$ and it crosses the continuum at $N = 100$ for the nucleus to ^{140}Zr . In this region the chemical potential is very small and almost parallel to the continuum limit. This means that the additional neutrons are added with a very small, nearly vanishing binding energy at the edge of the continuum. The total binding energies E for the isotopes above ^{122}Zr are therefore almost identical. This has been recognized already in Ref. [4] in RMF calculations using the BCS approximation and an expansion in an oscillator basis, which is definitely not reliable for chemical potentials so close to the continuum limit. In the present investigation we obtain the same result treating now the continuum and its coupling to the bound orbits by pairing correlations properly within full relativistic HB-theory in coordinate space.

To understand the kink in the neutron *rms*-radii shown in Fig. 1 we present in Fig. 2 the contributions $\langle r \rangle_{lj}$ of the different quantum numbers to these quantities as a function of the mass number. It is clearly seen that the radii for the negative parity levels close to the continuum limit are responsible for the rapid increase of the neutron radius. Neutrons above the closed neutron core $N = 82$ are filled into these orbits. As more and more neutron are added, $3p_{3/2}$ and $2f_{7/2}$ (after $N > 88$), $3p_{1/2}$ (after $N > 92$) respectively become weakly bound, then the contribution of further continuum $2f_{5/2}$ and $1h_{9/2}$ become more and more important. Going from $A = 122$ to $A = 140$ we observe an almost constant contribution of all the channels to the total *rms* matter radius except a sudden increase in the contribution of the $3p_{3/2}$, $2f_{7/2}$, $3p_{1/2}$ and $2f_{5/2}$ channels. This means that the giant halo in $^{124-140}\text{Zr}$ are formed by the occupation of all these levels in the respective nucleus.

In the upper panel of Fig. 3 we show the corresponding density distribution for neutrons and protons in the nucleus ^{134}Zr . By dashed lines we show calculations for different values of the box size $R=15, 20, 25$, and 30 fm. It is clear that we need very large box sizes to describe the halo properly. For $R=30$ fm the neutron density is reliably reproduced only up to $r=25$ fm, where it has decreased to 10^{-6} fm^{-3} . The full line in the upper panel of Fig. 3 is an asymptotic extension to infinite box size. On the other hand, the density distribution inside the nucleus is reproduced properly even for small values of the box size.

In the lower panel of Fig. 3 we show the relative contributions ρ_{nlj} of the different orbits characterized by the quantum numbers nlj with respect to the total neutron density ρ_n . For comparison we also show the total neutron density in the shaded area in arbitrary units. As we see the halo is formed essentially by contributions from three orbits $3p_{3/2}$, $3p_{1/2}$, and $2f_{7/2}$. The most inner part of the halo ($7 \leq r \leq 9$ fm) the $2f_{7/2}$ orbit plays the dominant role. As can be seen in the lower part of Fig. 1, this orbit is slightly below the chemical potential and to the continuum limit in this nucleus. Further outside ($10 \leq r \leq 15$ fm) its relative contribution is strongly reduced because of the larger centrifugal barrier felt by the $l=3$ orbit. In this region the orbit $3p_{3/2}$, which has nearly the same position as the $2f_{7/2}$ orbit, takes over. Because of the smaller orbital angular momentum, this orbit feels a reduced centrifugal barrier. For even larger distances from the center ($r \geq 15$ fm) its relative contribution is somewhat reduced and the $3p_{1/2}$ orbit gains importance. The $3p_{3/2}$ and the $3p_{1/2}$ levels feel the same centrifugal barrier, but the latter is situated directly at the continuum limit and therefore it is more loosely bound than the other two orbits.

In Fig. 4 we show for all the Zr-isotopes between $A=108$ and $A=140$ the occupation probabilities in the canonical basis of all the neutron levels near the Fermi surface, i.e. in the interval $-20 \leq E \leq 10$ MeV. The chemical potential is indicated by a vertical line. For the mass numbers $A < 122$ the chemical potential lies several MeV below the continuum limit ($E=0$) and there is only very little occupation in the continuum ($E > 0$). The nucleus ^{122}Zr has a magic neutron number and no pairing. As the neutron number goes beyond this closed core, the occupation of the continuum becomes more and more important. Adding up the occupation probabilities v^2 for the levels with $E > 0$ we find in the continuum 2 particles for $N=84$, 4 for $N=86$, 6 for $N=88$, roughly 3 for $N=90$, roughly 4 for $N=92$, roughly 3 for $N=94$, roughly 4 for $N=96$, roughly 5 for $N=98$, and roughly 6 for $N=100$ (where the neutron drip line is reached).

Summarizing our investigations, we predict neutron halos in the Zr nuclei close to the neutron drip line, which are composed not only by one or two neutrons, as is the case in the halos investigated so far in light p -shell nuclei, but which contain up to 6 neutrons. This is a new phenomenon, which has not been observed experimentally

so far. It would allow the study of collective phenomena in neutron matter of low density. This prediction is based on relativistic Hartree-Bogoliubov theory in the continuum. It combines the advantages of a proper description of the spin orbit term with those of full Hartree-Bogoliubov theory in the continuum, which allows in the canonical basis the scattering of Cooper pairs to low lying resonances in the continuum. A density dependent force of zero range has been used in the pairing channel. It contains no free parameter, because its strength is adjusted for the isotope ^{116}Zr to a similar calculation with Gogny's force D1S in the pairing channel. The halos are formed by two to six neutrons scattered as Cooper-pairs mainly to the levels $3p_{3/2}$, $2f_{7/2}$, $3p_{1/2}$, and $2f_{5/2}$. This is made possible by the fact that these resonances in the continuum come down very close to the Fermi level in these nuclei and by their coupling with the loosely bound levels just below the continuum limit.

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Figure Captions

Fig. 1 Upper part: r_{ms} -radii for neutrons and protons in Zr isotopes close to the neutron drip line as a function of the mass number A . Lower part: single particle energies for neutrons in the canonical basis as a function of the mass number. The dashed line indicates the chemical potential.

Fig. 2 Contributions $\langle r \rangle_{j\pi}$ for the different channels with the quantum numbers l and parity as a function of the mass. The left part show the orbits with positive parity and the right part shows the levels with negative parity.

Fig. 3 Upper part: neutron and proton density distribution in ^{134}Zr . Dashed lines indicate calculations for different values of the box size R . Lower part: relative contributions of the different orbits to the full neutron density as a function of the radius. The shaded area indicates the total neutron density in arbitrary units.

Fig. 4 The occupation probabilities in the canonical basis for various Zr isotopes with mass number A as a function of the single particle energy. The chemical potential is indicated by a vertical line. For $A > 122$ we also show the number N_h of neutrons in the halo.







